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FLUTTER OF CURVED AND FLAT SANDWICH PANELS SUBJECTED TO SUPERSONIC FLOW

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SUMMARY

A small-deflection theory is used in conjunction with a Galerkin technique to determine the flutter characteristics of simply supported sandwich panels, both flat and slightly curved. The sandwich cover plates are assumed to be of the same material and thickness and the core material is assumed isotropic. The flow is assumed supersonic over one surface, and static aerodynamic strip theory is used to describe the aerodynamic loading. A two-mode solution is used to illustrate the effects of curvature, core transverse shear stiffness, and midplane loads. It is found that the effects of curvature can be separated from the core stiffness effects.

INTRODUCTION

Considerations of dynamic aeroelastic instability, or flutter, of skin panels are important in the design of supersonic and hypersonic vehicles. A substantial body of literature treats various aspects of the panel flutter problem. (See refs. 1 to 5 and references cited therein.) Previous work has focused attention primarily on homogeneous isotropic panels. The sandwich panel, on the other hand, is being utilized increasingly in the design of aerospace vehicles. Under most circumstances, sandwich construction provides higher strength-weight ratios in a structure than a homogeneous panel and may provide suitable protection against micrometeoroid penetration. The primary distinction between sandwich and homogeneous panels is the greater importance of transverse shear flexibility in the sandwich panel (ref. 6). The purpose of the present investigation is to provide some estimate of the effect of transverse shear stiffness on the flutter boundaries for flat and curved sandwich panels. In this investigation both face sheets of the sandwich panel are assumed to be of the same material and the same thickness and the core material is assumed isotropic. The flow is assumed supersonic over one surface, and linearized static aerodynamic strip theory is used to describe the aerodynamic loading.

An equilibrium equation governing the lateral deflection of a curved sandwich panel is derived in reference 7 on the basis of small-deflection theory. This equation with modifications to account for the lateral inertia and aerodynamic loadings is used in this analysis. Effects of midplane loads, transverse shear stiffness, and panel curvature are evaluated.

SYMBOLS

$A = R_x - 2\nu^2$

a	length of panel, in.
$B = \dfrac{k^2}{\pi^4} + \nu^2 R_y - \nu^4$	
b	width of panel, in.
C	curvature parameter, $\dfrac{2t_s E_s a^4}{D_s r^2 \pi^4}$
D_Q	transverse shear stiffness of isotropic sandwich panel, lb/in.
D_s	flexural stiffness of isotropic sandwich panel, $\dfrac{E_s t_s^3}{6\left(1 - \mu^2\right)} + \dfrac{E_s t_s h^2}{2\left(1 - \mu^2\right)}$, in-lb
E_s	Young's modulus for faces of sandwich panel, lb/sq in.
h	depth of sandwich panel measured between middle surfaces of faces, in.
j,m,n,p,f	integers
$K_{jn} = j^2(j + 1)^2 + \nu^2\left(2j^2 + 2j + 1\right) + \nu^4$	
$k^2 = \gamma a^4 \omega^2 / D_s$	
$L_{jn} = \left(2j^2 + 2j + 1\right) + 2\nu^2$	
l	lateral aerodynamic loading, lb/sq in.
M	Mach number
M_{mp}	aerodynamic coupling parameter
N_x, N_y	midplane force intensities, lb/in.

Q	core stiffness parameter, $\frac{D_s \pi^2}{a^2 D_Q}$
q	dynamic pressure, $\frac{\rho U^2}{2}$, lb/sq in.
$R_x = \frac{N_x a^2}{\pi^2 D_s}$	
$R_y = \frac{N_y a^2}{\pi^2 D_s}$	
r	radius of curvature, in.
t	time, sec
t_s	thickness of sandwich face plates, in.
$t_l = 2t_s$	
U	velocity of airflow, in./sec
w	lateral deflection of panel, in.
w_{mn}	amplitude of mn mode, in.
x,y,z	Cartesian coordinates
$\beta = \sqrt{M^2 - 1}$	
γ	mass per unit area of panel, lb-sec^2/in.^{3}
λ	dynamic-pressure parameter, $\frac{2qa^3}{D_s \beta}$
λ_{cr}	critical value of λ
μ	Poisson's ratio
$\nu = \frac{na}{b}$	
ρ	mass density of air, lb-sec^2/in.^{4}
ω	circular frequency, radians/sec

A comma followed by a subscript denotes differentiation of the primary symbol with respect to the subscript.

STATEMENT OF THE PROBLEM

The configuration to be analyzed consists of a simply supported curved sandwich panel of uniform radius of curvature r, mounted in a rigid wall with air flowing over the top surface at Mach number M. (See fig. 1.) The face plates are of the same material and are of constant thickness t_s. The core has a uniform thickness $h - t_s$, where h is the distance between the middle surfaces of the face plates. The panel has a length a and a width b and is subjected to constant midplane force intensities N_x and N_y (positive in compression). The in-plane shear force intensity is assumed to be zero.

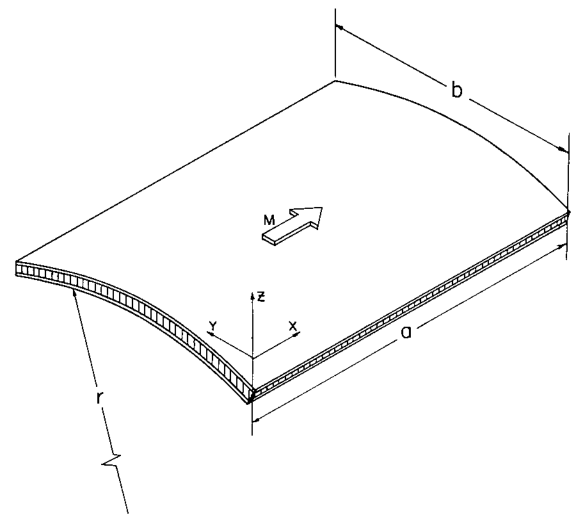

Figure 1.- Panel and coordinate system.

The equilibrium equation and appropriate boundary conditions are as follows (see refs. 1 and 7):

$$D_s\nabla^4 w + \left(1 - \frac{D_s}{D_Q}\nabla^2\right)\left(\frac{2t_sE_s}{r^2}\nabla^{-4}\frac{\partial^4 w}{\partial x^4} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + \gamma\frac{\partial^2 w}{\partial t^2} - l(x,y,t)\right) = 0 \tag{1a}$$

$$w(x,0,t) = w(x,b,t) = w(0,y,t) = w(a,y,t) = 0 \tag{1b}$$

$$w_{,yy}(x,0,t) = w_{,yy}(x,b,t) = w_{,xx}(0,y,t) = w_{,xx}(a,y,t) = 0 \tag{1c}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}$$

The inverse operator ∇^{-4} is defined by $\nabla^{-4}\left(\nabla^4 w\right) = \nabla^4\left(\nabla^{-4} w\right) = w$. The remaining terms in equation (1a) are defined as follows:

E_s Young's modulus for face plates

D_Q transverse shear stiffness of panel

D_s flexural stiffness of isotropic sandwich panel

γ mass of panel per unit area

$w(x,y,t)$ lateral deflection of panel

$l(x,y,t)$ lateral loading per unit area due to aerodynamic pressures

For static strip theory, the lateral loading is given by the simple Ackeret value

$$l(x,y,t) = -\frac{2q}{\beta}\frac{\partial w}{\partial x} \tag{2}$$

where q is the dynamic pressure $\rho U^2/2$, and $\beta = \sqrt{M^2 - 1}$. The development of equation (1a) is based on the assumptions that the deflections are small compared to the total panel thickness, the total thickness is small compared to the radius of curvature, and the sandwich core carries no direct stress.

ANALYSIS

A solution which satisfies the boundary conditions for simply supported edges is assumed as follows:

$$w(x,y,t) = \sum_{m=1}^{f} w_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega t} \tag{3}$$

where w_{mn} is the amplitude of the mn mode and ω is the circular frequency. The frequency ω is, in general, complex; however, attention is directed primarily to real values for which the motion is harmonic. Substituting equation (3) into equation (1a) and nondimensionalizing by multiplying by $\frac{a^4}{D_s\pi^4}$ results in the following equation:

$$\sum_{m=1}^{f}\left\{\left[1 + \frac{C}{\left(m^2 + \nu^2\right)^2} + \frac{CQ}{\left(m^2 + \nu^2\right)} - QR_x\right]m^4 - \left(R_x + QR_x\nu^2 - 2\nu^2\right.\right.$$

$$\left.\left. + Q\nu^4 + QB\right)m^2 - \left(B + QB\nu^2 + Q\nu^6\right)\right\} w_{mn} \sin\left(\frac{m\pi x}{a}\right)$$

$$+ \sum_{m=1}^{f} \frac{\lambda m}{\pi^3}\left(Qm^2 + Q\nu^2 + 1\right)w_{mn} \cos\left(\frac{m\pi x}{a}\right) = 0 \tag{4}$$

where

$$C = \frac{2t_s E_s a^4}{D_s r^2 \pi^4}$$

$$Q = \frac{D_s \pi^2}{D_Q a^2}$$

$$R_x = \frac{N_x a^2}{\pi^2 D_s}$$

$$R_y = \frac{N_y a^2}{\pi^2 D_s}$$

$$\lambda = \frac{2qa^3}{D_s \beta}$$

$$k^2 = \frac{\gamma a^4 \omega^2}{D_s}$$

$$\nu = \frac{na}{b}$$

$$B = \frac{k^2}{\pi^4} + \nu^2 R_y - \nu^4$$

The quantities C and Q are dimensionless curvature and core stiffness parameters, respectively; R_x and R_y are midplane loading parameters; λ is a dynamic-pressure parameter; k^2 is a frequency parameter; and ν is a length-width-ratio parameter.

Applying the Galerkin technique by multiplying equation (4) by $\sin\frac{p\pi x}{a}$ and integrating yields the following set of equations for the coefficients w_{mn}:

$$\left\{\left[1 + \frac{C}{\left(m^2 + \nu^2\right)^2} + \frac{CQ}{\left(m^2 + \nu^2\right)} - R_x Q\right]m^4 - \left(R_x - 2\nu^2 + QR_x\nu^2 + QB + Q\nu^4\right)m^2 - \left(B + QB\nu^2 + Q\nu^6\right)\right\}w_{mn} - \sum_{p=1}^{f}\frac{\lambda M_{mp}}{\pi^4}\left(Qp^2 + Q\nu^2 + 1\right)w_{pn} = 0$$

$$(m = 1, 2, \ldots, f) \qquad (5)$$

where

$$M_{mp} = \begin{cases} \dfrac{4mp}{p^2-m^2} & \text{where } p+m \text{ is odd} \\ 0 & \text{where } p+m \text{ is even} \end{cases}$$

It can be noted that equations (4) and (5) are uncoupled in the cross-stream direction; n is only a parameter of the solution.

A general two-mode solution is pursued corresponding to any two consecutive modes j and $j + 1$, and a characteristic equation is obtained by setting the determinant of the coefficients of equations (5) equal to zero. By use of the definition of flutter discussed in reference 1, the characteristic equation is solved for λ, and λ is maximized with respect to the frequency parameter B to give a critical value λ_{cr}, which is a measure of the flutter speed. This procedure results in the following closed-form expression for λ_{cr}:

$$\lambda_{cr} = \frac{(2j + 1)^2\pi^4}{8j(j + 1)}\left\{-R_x + \frac{K_{jn}Q + L_{jn}}{K_{jn}Q^2 + L_{jn}Q + 1} + \frac{C\left[2j^2(j + 1)^2\nu^2 + \left(2j^2 + 2j + 1\right)\nu^4\right]}{K_{jn}{}^2}\right\} \qquad (6)$$

where

$$K_{jn} = j^2(j + 1)^2 + \nu^2\left(2j^2 + 2j + 1\right) + \nu^4$$

$$L_{jn} = \left(2j^2 + 2j + 1\right) + 2\nu^2$$

A solution containing two consecutive modes, j and $j + 1$, is assumed since, in the region of interest, the minimum flutter speed occurs with the coalescence of the frequencies of consecutive modes. More accurate solutions can be obtained by including more modes or by solving the uncoupled differential equation exactly with n as a parameter. The relative accuracy of these methods as applied to a flat homogeneous isotropic plate is shown in reference 1.

If $j = 1$, equation (6) gives the following two-term approximation for λ_{cr} corresponding to the first two modes in the streamwise direction and any one mode in the cross-stream direction:

$$\lambda_{cr} = \frac{9\pi^4}{16}\left[-R_x + \frac{K_{1n}Q + L_{1n}}{K_{1n}Q^2 + L_{1n}Q + 1} + \frac{C\left(8\nu^2 + 5\nu^4\right)}{K_{1n}^2}\right] \tag{7}$$

where

$$K_{1n} = 4 + 5\nu^2 + \nu^4$$

$$L_{1n} = 5 + 2\nu^2$$

LIMITING CASES

If the core thickness of the sandwich panel $h - t_s$ is allowed to approach zero as D_Q approaches infinity and if the thickness and flexural stiffness of the resulting homogeneous isotropic panel are denoted by t_l and D, respectively, then

$$h = t_s$$

$$t_l = 2t_s$$

$$Q = 0$$

$$D_s = \frac{E_s t_l^3}{12(1 - \mu^2)} = D$$

$$C = \frac{12a^4(1 - \mu^2)}{\pi^4 t_l^2 r^2}$$

and equation (7) reduces to

$$\lambda_{cr} = \frac{9\pi^4}{16}\left[(5 - A) + \frac{12a^4(1 - \mu^2)(5\nu^4 + 8\nu^2)}{\pi^4 t_l^2 r^2 (4 + 5\nu^2 + \nu^4)^2}\right] \tag{8}$$

where $A = R_x - 2\nu^2$. Equation (8) corresponds to equation 1.21 in reference 2 (where m and r in equation 1.21 are equal to 1 and 2, respectively) for the flutter of a curved homogeneous panel. Further, by letting the panel radius r approach infinity, equation (8) becomes

$$\lambda_{cr} = \frac{9\pi^4}{16}(5 - A) \tag{9}$$

which corresponds to equation (16) in reference 1 for the flutter of a flat homogeneous panel.

RESULTS AND DISCUSSION

Flat Sandwich Panel

The critical dynamic-pressure parameter λ_{cr} for a flat sandwich panel can be obtained from equation (6) by letting $C = 0$. Equation (6) then becomes

$$\lambda_{cr} = \frac{(2j + 1)^2 \pi^4}{8j(j + 1)}\left(-R_x + \frac{K_{jn}Q + L_{jn}}{K_{jn}Q^2 + L_{jn}Q + 1}\right) \tag{10}$$

The value of λ_{cr} in equation (10) is a function of R_x, j, ν, and Q. For a given value of Q and a moderate value of a/b, the flutter boundary $\left(\lambda_{cr}\right.$ as a function of $\left.R_x\right)$ is a minimum for $j = n = 1$, for the range of R_x considered herein.

To illustrate the effects of the core stiffness parameter Q, a value of a/b of 1 is chosen. For $j = \nu = 1$, equation (10) becomes

$$\lambda_{cr} = \frac{9\pi^4}{16}\left(-R_x + \frac{10Q + 7}{10Q^2 + 7Q + 1}\right) \tag{11}$$

The flutter boundaries $\left(\lambda_{cr}\right.$ as a function of $\left.R_x\right)$ for various values of Q are presented in figure 2. The boundary for $Q = 0$ is the boundary for a flat homogeneous panel of thickness $2t_s$. The exact solution for a flat homogeneous panel is also shown in figure 2. (See ref. 1.) The flutter boundary is very sensitive to the value of Q, especially in the lower ranges of Q $(0 < Q < 1)$. This sensitivity is illustrated in figure 3 in which λ_{cr} is plotted as a function of Q for various values of R_x. The steep slopes of these curves for $0 < Q < 1$ indicate the large influence that the core stiffness has on the flutter boundary in this range. The range of validity of the results in figures 2 and 3 is limited by the buckling characteristics of the panel. For example, a square panel with no stress in the y-direction $\left(R_y = 0\right)$ and $Q = 0.1$ buckles if $R_x = 3.3$ (for $\lambda = 0$). In this case the curve for $Q = 0.1$ in figure 2 is only valid for R_x up to approximately 3.3 (see ref. 3). Other values of a/b and R_y yield other ranges of validity.

It should be noted that all parameters considered in figure 2 $\left(\lambda_{cr},\ Q,\right.$ and $\left.R_x\right)$ are functions of the panel flexural stiffness D_s. As a result, a

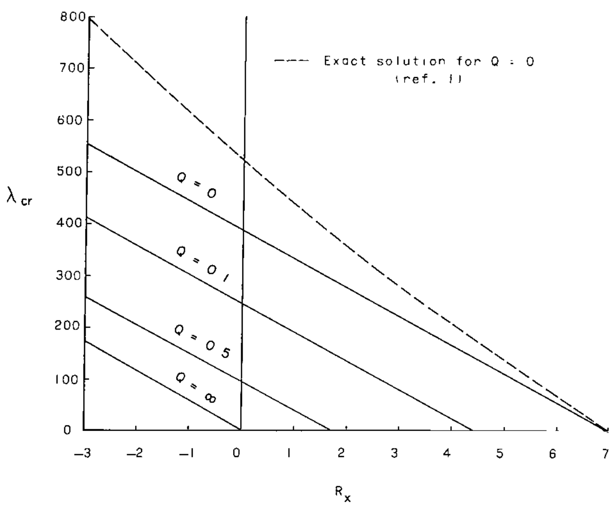

Figure 2.- Effect of transverse shear stiffness on flutter boundary for flat sandwich panel. $C = 0$; $j = \nu = 1$.

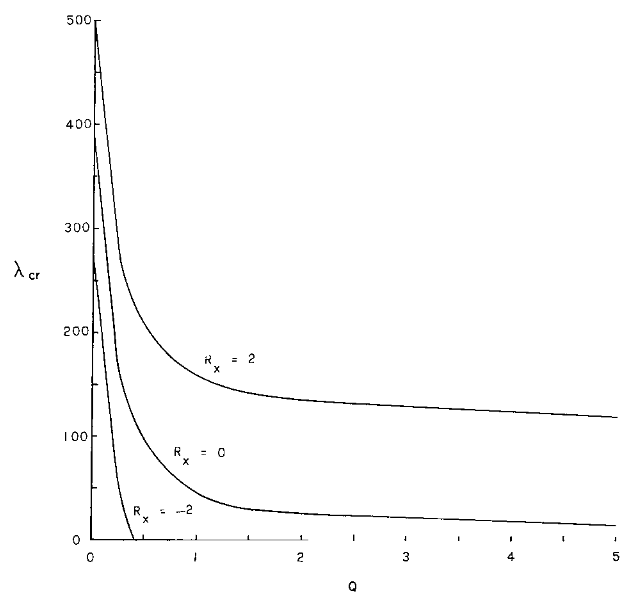

Figure 3.- Variation of flutter parameter with transfer shear stiffness for several values of streamwise in-plane load parameter. $\nu = 1$; $R_y = 0$; $C = 0$.

sandwich panel having the same flutter speed as a homogeneous panel provides a reduction in weight that is not obvious from figure 2. For example, a typical flat honeycomb sandwich panel with cover plates and core of the same material, a core density of 3 percent of the material density, $\frac{h}{t_s} = 10$, a length-width ratio a/b of 1, length a of 20, core stiffness parameter Q of 0.0521, and cover plate thickness t_s of 0.0441 inch flutters at the same dynamic pressure q as a homogeneous panel of the same material and a thickness of 0.342 inch. The weight of the sandwich panel, however, is only 29 percent of the weight of the homogeneous panel.

Curved Sandwich Panel

Flutter boundaries for a curved sandwich panel are obtained from equation (6). The effect of curvature on λ_{cr} is given by the expression

$$\Delta\lambda_{cr} = \frac{(2j + 1)^2\pi^4}{8j(j + 1)} C \frac{\left[2j^2(j + 1)^2\nu^2 + \left(2j^2 + 2j + 1\right)\nu^4\right]}{K_{jn}^{\ 2}}$$

which is the same correction factor derived in reference 2. The curvature effects are entirely contained in this expression and are completely separated from the effects of the core stiffness parameter Q. For this reason, the effects of curvature can be thought of in the form of a correction term which can be added to the value of λ_{cr} for a flat sandwich panel, as is done in reference 2 for homogeneous panels.

One important characteristic of curved panels is that, whereas a flat panel has a minimum flutter boundary for $j = n = 1$ for the range of R_x considered herein, a curved panel can have a minimum boundary with other combinations of j and n. As a result, various combinations of j and n must be examined by trial and error in order to determine the combination that results in the minimum flutter boundary. A more detailed discussion on the flutter of curved panels is given in reference 2. Although the discussion in reference 2 is concerned specifically with homogeneous panels, the essential features also apply to sandwich panels.

Figure 4 is a plot of the flutter boundaries for curved sandwich panels for various values of the core stiffness parameter Q and for specific values of the curvature parameter and length-width ratio $\left(C = 100; \frac{a}{b} = 1\right)$. The boundaries have been minimized (by trial and error) with respect to j and n and are, therefore, the lowest boundaries for the range of R_x considered. The flutter boundary is greatly affected by the core stiffness in much the same manner as it was for the flat panel. Flutter boundaries for curved sandwich panels $\left(Q = 0.1 \text{ and } \frac{a}{b} = 1\right)$ are shown in figure 5 for various values of C.

The boundaries have again been minimized with respect to j and n. It can be seen from this figure that curvature tends to stiffen the panel and raise the flutter boundary.

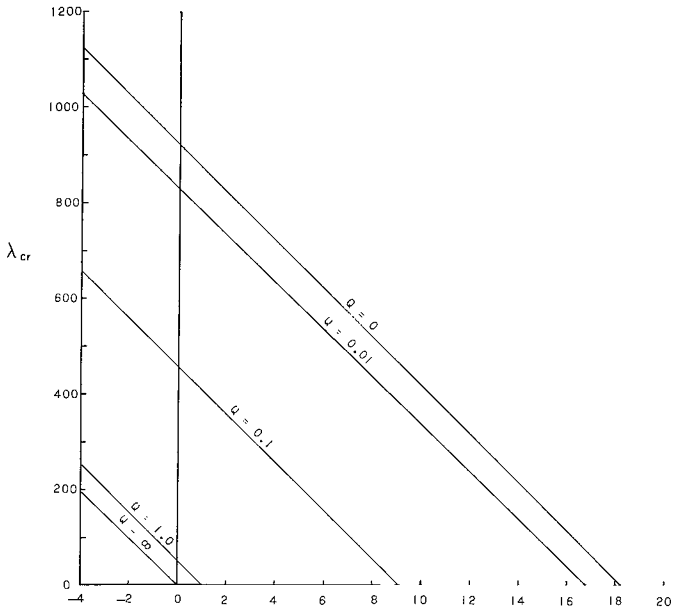

Figure 4.- Effect of transverse shear stiffness on flutter boundary for curved sandwich panel. $C = 100$; $\frac{a}{b} = 1$.

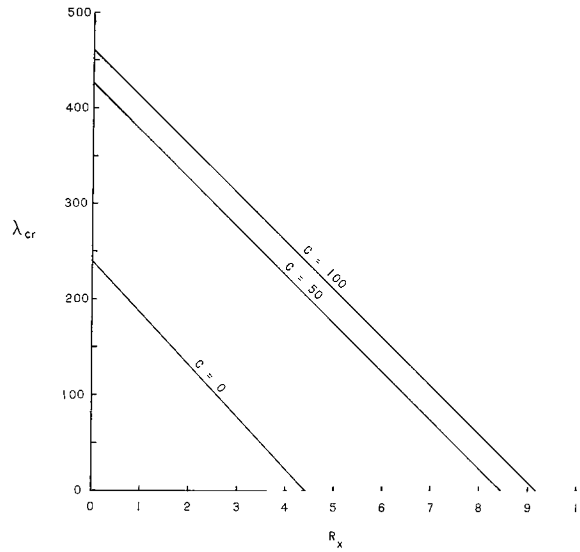

Figure 5.- Effect of curvature on flutter boundary for sandwich panel. $\frac{a}{b} = 1$; $Q = 0.1$.

CONCLUDING REMARKS

A small-deflection theory is used in conjunction with a two-mode Galerkin solution to analyze approximately the flutter of sandwich panels, both flat and slightly curved. Closed-form expressions are obtained from which flutter boundaries can be determined. For flat sandwich panels the flutter boundaries are determined in a straightforward manner, but for curved sandwich panels the determination of boundaries requires a trial and error minimization process.

For a given application and flutter speed a sandwich panel can provide a substantial reduction in weight as compared with a homogeneous panel. In general, the effect of core transverse shear stiffness on the flutter characteristics of sandwich panels is found to be significant. Curvature effects are found to be completely separated from core stiffness effects and within the limitations of the present analysis can be treated in the same way as curvature of homogeneous panels.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., November 22, 1963.

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